International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 4, Issue 2, pp: (1-6), Month: October 2016 - March 2017, Available at: www.researchpublish.com

# The Study of Quantum Dynamics of Neutral Cold Atom in Different External Field Configurations

Pandeng Shi

School of Science, Department of Physics, Tianjin Polytechnic University, Tianjin 300387, China

*Abstract:* **In this article, we study the problem of quantum dynamics with neutral cold atom in two different kinds of external field configurations. We demonstrate in the presence of a configuration of crossed radial electric and uniform magnetic fields or in the presence of a configuration of crossed radial magnetic and uniform electric fields the induced electric dipoles in a system of neutral cold atoms have a quantization similar to the Landau Levels. The energy levels and eigen functions are obtained.**

*Keywords:* **cold atom, electric dipole moment, field configuration.**

## I. INTRODUCTION

The study of the quantum dynamics of charged and neutral particles in the presence of electromagnetic fields is responsible for a series of geometrical and topological effects in physics. The interaction of an electromagnetic field with a charged particle, plays important role in the generation of collective phenomena, for instance, fractional statistics [1] and the quantum Hall effect [2]. The quantum motion of a charged particle in the presence of a constant magnetic field is described by Landau theory. The Landau quantization in two dimensions makes the energy levels coalesce into a discrete spectrum. The Landau levels present a remarkable interest from many points of view. It is the simplest model necessary for the description of the quantum Hall effect. On the order hand, the Landau levels were studied for different curved surfaces [3, 4] with the interest in several areas of physics. The idea of analogs of Landau quantization was proposed initially by Ericsson and Sjörvist [5] inspired in the work of Paredes et al [6, 7] that studied the possibility of an analog of the Hall effect in Bose-Einstein condensates. The idea of Ericsson and Sjörvist was that the Aharonov-Casher (AC) interaction can be used to generate an analog of Landau levels in systems of neutral atoms. They used the AC interaction, for a neutral particle with a permanent magnetic dipole, and proposed the analogy with the Landau levels quantization, for certain field-dipole configurations, that they have denominated of Landau-Aharonov-Casher (LAC) levels. Result is interesting and suggests the possibility of quantum Hall effect for magnetic dipoles in the presence of an electric field. In recent years the advance of cold atom technology made it possible to simulate several solid state effects employing neutral atoms and techniques from quantum optics [8–10]. Recently, the neutral atoms techniques to simulate the behaviour of a charged particle in this systems [11, 12] has been developed. In this way, the study of systems that simulate the strongly interacting system in a cold atom has attract much attention in recent years [13, 14]. In the present work, Claudio Furtado [15] has studied the induced electric dipole which has a exact dipole moment in a radial electric field crossed with uniform magnetic field configuration and has given the energy levels and eigenfunctions. However, he’s work is not comprehensive. In this work, we will analyze the quantum dynamics problem with an induced electric dipole in two different kinds of external electromagnetic field configurations. This field configuration confines the dipole in a plane and produces a coupling similar to the coupling of a charged particle in the presence of external magnetic field. In Sect. II, we briefly outline the Claudio Furtado’ work and give analyses about the energy levels. Based on the present work we will emphatically study the properties of an induced electric dipole in a new electromagnetic field configuration in Sect. III. In this letter we adopt the systems of unity were $\hbar = c = 1$.

## II. COLD ATOM IN THE PRESENCE OF A CONFIGURATION OF CROSSED RADIAL ELECTRIC AND UNIFORM MAGNETIC FIELDS

The cold atom is treated as structureless induced dipole moment. We consider that our system was submitted the following radial electric field

$$\mathbf{E} = \frac{\rho}{2} r\hat{e}_r, \tag{1}$$

where $\rho$ is the charge density. The system is also submitted to the external uniform magnetic field

$$\mathbf{B} = B_0\hat{e}_z. \tag{2}$$

The presence of electromagnetic field induces an electric dipole moment in the cold atom given by

$$\mathbf{d} = \kappa(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{3}$$

where $\kappa$ is the dielectric polarizability of the cold atoms and $\mathbf{v}$ is the velocity of the atom. The Lagrangian in the presence of the electromagnetic field described in the paragraph above, in the nonrelativistic limit, is given by[16]

$$L = \frac{1}{2}Mv^2 + \frac{1}{2}\mathbf{d} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{4}$$

where M is the mass of cold atom. Using the dipole moment given by (3) we obtain the following Lagrangian in cold atoms

$$L = \frac{1}{2}(M + \kappa B^2)v^2 + \frac{1}{2}\kappa E^2 + \kappa\mathbf{v} \cdot \mathbf{B} \times \mathbf{E}. \tag{5}$$

According to the relationship of the translation between Hamiltonian and Lagrangian $H = -L + \sum p_i v_i$ we can get the Hamiltonian of the system

$$H = \frac{1}{2m^*}[\mathbf{P} + \kappa(\mathbf{E} \times \mathbf{B})]^2 - \frac{1}{2}\kappa E^2, \tag{6}$$

where $m^* = M + \kappa B^2$ and $\mathbf{P} = m^*\mathbf{v} + \kappa\mathbf{B} \times \mathbf{E}$. The Hamiltonian (6) presents an analogy to the minimal coupling for a charged particle in the presence of the magnetic field. We can define then the effective potential vector as being $\mathbf{A}_{eff} = \mathbf{E} \times \mathbf{B}$. Using the field configuration given in (1) and (2), we obtain the following effective vector potential

$$\mathbf{A}_{eff} = -\frac{1}{2}B_0\rho r\hat{e}_\phi. \tag{7}$$

Using the definition above we can go on to define the effective magnetic field associated with the effective vector potential as

$$\mathbf{B}_{eff} = B_0\rho\hat{e}_z. \tag{8}$$

Note that this configuration of effective magnetic field is uniform. This configuration with the dipole configuration and movement of dipole restricted to the plane satisfies the conditions demonstrated by Ericsson and Sjörvist to obtain the analogous of Landau Levels, that are: $\mathbf{B}_{eff}$ uniform, absence of torque on the dipole and electrostatic conditions $\frac{\partial E}{\partial t} = 0$ and $\nabla \times \mathbf{E} = 0$. All these conditions are satisfied by the configuration presented here that are similar to configurations presented in ref. [17]. In this way we write the stationary Schrödinger equation for this system, in cylindrical coordinates, in the following form

$$-\frac{1}{2m^*}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}\right]+\frac{i\omega}{2}\frac{\partial\psi}{\partial\phi}+\frac{m^*\omega^2}{8}r^2\psi-\frac{m^{*2}\omega^2}{8\kappa B_0^2}r^2\psi=\varepsilon\psi, \tag{9}$$

where $\omega=\frac{\kappa B_0\rho}{m^*}$. We use the following Ansatz to the solution of Scrhrödinger equation

$$\psi=Ce^{i\ell\phi}R(r), \tag{10}$$

where $\ell$ is an integer number and C is a normalization constant. Using Eq. (10), Eq. (9) assumes the following form:

$$\frac{1}{2m^*}\left(R''(r)+\frac{1}{r}R'(r)-\frac{\ell^2}{r^2}R(r)\right)+\left(\varepsilon+\frac{\ell\omega}{2}-\frac{m^*\omega^2}{8}r^2-\frac{m^{*2}\omega^2}{8\kappa B_0^2}r^2\right)R(r)=0. \tag{11}$$

Now, We use the following change of variables

$$\xi=\frac{m^*\omega}{2}r^2. \tag{12}$$

In this way, Eq. (11) can be transformed into

$$\xi R''(\xi)+R'(\xi)+\left(-\frac{\delta\xi}{4}+\beta-\frac{\ell^2}{4\xi}\right)R(\xi)=0, \tag{13}$$

where $\beta=\frac{\varepsilon}{\omega}-\frac{\ell}{2}$ and $\delta=1+\frac{m^*}{\kappa B_0^2}$. Assuming, for the radial eigenfunction, the form

$$R(\xi)=e^{-\delta\xi/2}\xi^{|\ell|/2}\zeta(\xi), \tag{14}$$

We obtain a degenerated hypergeometric equation that is satisfied by the function $\zeta(\xi)$ given by

$$\zeta(\xi)=F[-\gamma,|\ell|+1,\delta\xi], \tag{15}$$

where $\gamma=\beta-\frac{\delta(|\ell|+1)}{2}$. Then, the energy is given by

$$\varepsilon_{n,\ell}=\left(n+\frac{|\ell|}{2}+\frac{1}{2}\right)\delta\omega+\frac{\ell}{2}\omega, \tag{16}$$

where n is an integer number. In the limit that $\delta\to 1$ the eigenvalues are given by

$$\varepsilon_{n,\ell}=\left(n+\frac{|\ell|}{2}+\frac{1}{2}+\frac{\ell}{2}\right)\omega. \tag{17}$$

Note that, this limit is characterized by a high magnetic field. In this case the system is similar to Landau levels of a charged particle. The radial eigenfunction is then given by

$$R_{n,\ell}=\frac{1}{a^{1+|\ell|}}\left[\frac{(|\ell|+n)!}{2^{|\ell|}n!\,|\ell|!^2}\right]^{1/2}\exp\left(-\frac{\delta r^2}{4a^2}\right)\times r^{|\ell|}F\left[-n,|\ell|+1,\frac{\delta r^2}{2a^2}\right], \tag{18}$$

where $a=\sqrt{\frac{1}{\kappa\rho B_0}}$. In this section we study the eigenfunctions and the eigenvalues of a neutral cold atom with an induced dipole moment in the presence of crossed radial electric and uniform magnetic fields. The field configuration is the same of Claudio Furtado [15] and confines particles in two dimensions. In a strong magnetic field, the energy levels are similar to Landau levels. Based in this fact the possibility of an atomic analog of the Landau quantization to electric dipole is presented in a similar way to Landau quantization investigation of the magnetic dipole [5]. This effect can be viewed as a first approach to investigate a atomic analog of quantum Hall effect with electric dipoles in cold atoms.

## III. COLD ATOM IN THE PRESENCE OF A CONFIGURATION OF CROSSED RADIAL MAGNETIC AND UNIFORM ELECTRIC FIELDS

Now, we concentrate in the analysis of a Landau levels analogue for the quantum dynamics of an electric dipole in the presence of crossed radial magnetic and uniform electric fields. The central idea of this letter is similar to the approach developed by Ericsson and Sjörvist to the LAC levels, presented previously in this letter, using a Schrödinger equation approach. In our study of this problem we demonstrate that in specific field-dipole configurations we have a quantization similar to Landau levels. We consider a radial magnetic field in the following form

$$\mathbf{B} = \frac{\rho_m}{2} r\hat{e}_r, \tag{19}$$

where $\rho_m$ is magnetic charge density. We can see clearly that this configuration is generated by a distribution of magnetic charge. The arrangement of field configurations with the radial magnetic field is more difficult to achieve experimentally, due to the fact that, we need a distribution of magnetic charges. We can observe in the literature that this kind of arrangement would be possible. Some authors have claimed that this configuration can be obtained experimentally as in the arrangements presented in the articles [18,19]. And our system was submitted the following electric field at the same time

$$\mathbf{E} = E_0\hat{e}_z. \tag{20}$$

Due to the induced electric dipole is invariable when we change the external field, we use the electric dipole given in (3).Then the nonrelativistic Hamiltonian that describes the quantum dynamics of the electric dipole, in the presence of the external field (19) and (20), is given by[16]

$$H = \frac{1}{2m^*}[\mathbf{P} + \kappa(\mathbf{E}\times\mathbf{B})]^2 - \frac{1}{2}\kappa E^2, \tag{21}$$

Next, we give the effective vector potential similar to Eq.(7) in section II

$$\mathbf{A}_{\mathrm{eff}} = \mathbf{E}\times\mathbf{B} = \frac{1}{2}\rho_m E_0 r\hat{e}_\phi. \tag{22}$$

Then we can get the effective magnetic field as

$$\mathbf{B}_{\mathrm{eff}} = \nabla\times\mathbf{A} = \rho_m E_0\hat{e}_z, \tag{23}$$

Note that κ plays the role of a coupling constant in (21). Here the same condition found by Ericsson and Sjörvist [5] is obeyed for the existence of an analogue of Landau levels. Using Eq. (23) we have that $\mathbf{B}_{\mathrm{eff}}$ is homogeneous.It obeys the necessary conditions for the existence of a analogue of Landau Levels. The stationary Schrödinger equation in cylindrical coordinates is

$$-\frac{1}{2m^*}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}\right] - \frac{i\omega}{2}\frac{\partial\psi}{\partial\phi} + \frac{m^*\omega^2}{8}r^2\psi - \frac{\kappa E_0^2}{2}\psi = \varepsilon\psi, \tag{24}$$

where $\omega = \frac{\kappa\rho_m E_0}{m^*}$. This equation is solved using the following Ansatz

$$\psi = Ce^{i\ell\phi}R(r), \tag{25}$$

where $\ell$ is an interger number and C is a normalization constant. Substitute this wave function (25) into the Schrödinger equation (24) we obtain the radial equation

$$\frac{1}{2m^*}\left(R''(r) + \frac{1}{r}R'(r) - \frac{\ell^2}{r^2}R(r)\right) + \left(\varepsilon + \frac{\kappa E_0^2}{2} - \frac{\omega\ell}{2} - \frac{m^*\omega^2}{8}r^2\right)R(r) = 0. \tag{26}$$

Now, by using the change of variable $\xi = \frac{m^*\omega}{2} r^2$ , Eq. (26) is transformed into

$$\xi R''(\xi) + R'(\xi) + \left(-\frac{\xi}{4} + \beta - \frac{\ell^2}{4\xi}\right) R(\xi) = 0, \tag{27}$$

where

$$\beta = \frac{\varepsilon}{\omega} - \frac{\ell}{2} + \frac{\kappa E_0^2}{2\omega}. \tag{28}$$

Assuming, for the radial eigenfunction, we can write the solution in the form

$$R(\xi) = e^{-\xi/2} \xi^{|\ell|/2} \zeta(\xi), \tag{29}$$

which satisfies the usual asymptotic requirements and the finiteness at the origin for the bound state, we have

$$\xi \frac{d^2\zeta}{d\xi^2} [(|\ell| + 1) - \xi] \frac{d\zeta}{d\xi} - \gamma\zeta = 0, \tag{30}$$

where $\gamma = \beta - \frac{|\ell|+1}{2}$. We find that the solution of equation (30) is the degenerated hypergeometric function

$$\zeta(\xi) = F[-\gamma, |\ell| + 1, \xi]. \tag{31}$$

In order to have normalization of the wavefunction, the series in (31) must be a polynomial of degree n, therefore

$$\gamma = \beta - \frac{|\ell| + 1}{2} = n, \tag{32}$$

where $n = 0, \pm 1, \pm 2$ ... With this condition, we obtain discrete values for the energy, given by

$$\varepsilon = \left(n + \frac{|\ell|}{2} + \frac{1}{2} + \frac{\ell}{2} - \frac{\kappa E_0^2}{2\omega}\right) \omega. \tag{33}$$

Note that, duo to the presence of electric field $\varepsilon$ given in (33) is continuous. If we want to quantize the value of $\varepsilon$ similar to Landau levels, only take the limit that $E_0 \to 0$ i.e. the cole atom in our system only be submitted the radial magnetic field. Then the eigenvalues are given by

$$\varepsilon_{n,\ell} = \left(n + \frac{|\ell|}{2} + \frac{1}{2} + \frac{\ell}{2}\right) \omega. \tag{34}$$

The radial eigenfunction is then given by

$$R_{n,\ell} = \frac{1}{a^{1+|\ell|}} \left[\frac{(|\ell| + n)!}{2^{|\ell|} n! \, |\ell|!^2}\right]^{1/2} \exp\left(-\frac{r^2}{4a^2}\right) \times r^{|\ell|} F\left[-n, |\ell| + 1, \frac{r^2}{2a^2}\right], \tag{35}$$

where $a = \sqrt{\frac{1}{\kappa \rho_m E_0}}$.

## IV. SUMMARY AND CONCLUSIONS

In this paper we study the eigenfunctions and the eigenvalues of a neutral particle (cold atom) with an induced dipole moment in the presence of two different kinds of external crossed electric and magnetic fields. The field configuration confines particles in two dimensions. In the presence of field configuration of Sect. II, we find that in a strong magnetic field, the energy levels are similar to Landau levels. On the contrary, in the Sect. III, the phenomenon of landau quantization can be found when the external electric field is very weak. Note that, in this work, we ignore the interaction between magnetic moment of cold atom system and external fields in the Hamiltonian given in (6) and (21). In conclusion, energy eigenfunctions and the eigenvalues of a neutral cold atom with a permanent electric dipole moment in the presence of two different kinds of external fields are obtained exactly. The energy levels are similar to Landau levels and they change follow with the external fields.

## REFERENCES

[1] Andrij Rovenchak. Complex-valued fractional statistics for d-dimensional harmonic oscillators. Physics Letters A, 378(3):100–108, 2014.

[2] Ahmed Jellal. Integer quantum hall effect in graphene. Physics Letters A, 380(17):1514–1516, 2016.

[3] James G. Analytis, Ross D. McDonald, Scott C. Riggs, Jiun-Haw Chu, G. S. Boebinger, and Ian R. Fisher. Two-dimensional surface state in the quantum limit of a topological insulator. Nat Phys, 6(12):960–964, 2010. 10.1038/nphys1861.

[4] H. Aoki and H. Suezawa. Landau quantization of electrons on a sphere. Physical Review A, 46(3):R1163–R1166, 1992. PRA.

[5] Marie Ericsson and Erik Sj¨oqvist. Towards a quantum hall effect for atoms using electric fields. Physical Review A, 65(1):013607, 2001. PRA.

[6] B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller. $\frac{1}{2}$anyons in small atomic bose-einstein condensates. Physical Review Letters, 87(1):010402, 2001. PRL.

[7] B. Paredes, P. Zoller, and J. I. Cirac. Fractional quantum hall regime of a gas of ultracold atoms. Solid State Communications, 127(2):155–162, 2003.

[8] A. B. Kuklov and B. V. Svistunov. Counterflow superfluidity of two-species ultracold atoms in a commensurate optical lattice. Physical Review Letters, 90(10):100401, 2003. PRL.

[9] Jiannis K. Pachos and Enrique Rico. Effective three-body interactions in triangular optical lattices. Physical Review A, 70(5):053620, 2004. PRA.

[10] L. M. Duan, E. Demler, and M. D. Lukin. Controlling spin exchange interactions of ultracold atoms in optical lattices. Physical Review Letters, 91(9):090402, 2003. PRL.

[11] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller. Cold bosonic atoms in optical lattices. Physical Review Letters, 81(15):3108–3111, 1998. PRL.

[12] D. Jaksch and P. Zoller. Creation of effective magnetic fields in optical lattices: the Hofstadter butterfly for cold neutral atoms. New Journal of Physics, 5(1):56, 2003.

[13] B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller. $\frac{1}{2}$anyons in small atomic bose-einstein condensates. Phys. Rev. Lett., 87:010402, Jun 2001.

[14] B. Paredes, P. Zoller, and J.I. Cirac. Fractional quantum hall regime of a gas of ultracold atoms. Solid State Communications, 127(2):155 – 162, 2003. Quantum Phases at the Nanoscale.

[15] Claudio Furtado, J. R. Nascimento, and L. R. Ribeiro. Landau quantization of neutral particles in an external field. Physics Letters A, 358(5C6):336–338, 2006.

[16] J. Anandan. Electromagnetic effects in the quantum interference of dipoles. Physics Letters A, 138(8):347 – 352, 1989.

[17] Marie Ericsson and Erik Sj¨oqvist. Towards a quantum hall effect for atoms using electric fields. Phys. Rev. A, 65:013607, Dec 2001.

[18] S Y Molokov and J E Allen. On the theory of the heiser and shercliff experiment. i. mhd flow in an open channel in a strong uniform magnetic field. Journal of Physics D: Applied Physics, 25(3):393, 1992.

[19] S Y Molokov and J E Allen. On the theory of the heiser and shercliff experiment. ii. Mhd flow between two cylinders in a strong radial magnetic field. Journal of Physics D: Applied Physics, 25(6):933, 1992.